

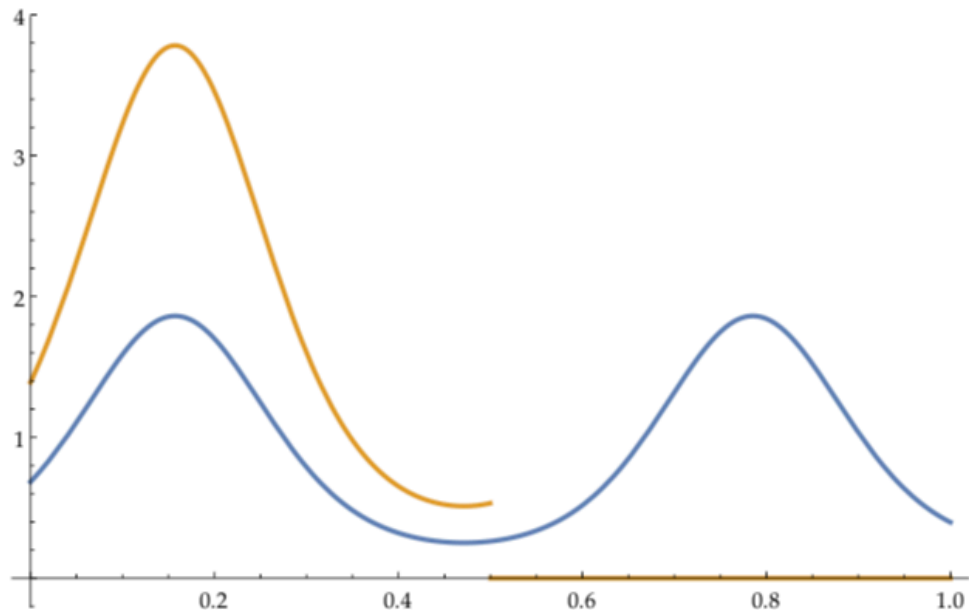
# Learning from Biased Data

Constantinos Daskalakis

EECS & CSAIL, MIT



# Amuse Bouche



$$f(x) = \frac{e^{\sin 10x}}{\int_0^1 e^{\sin 10x} dx}$$

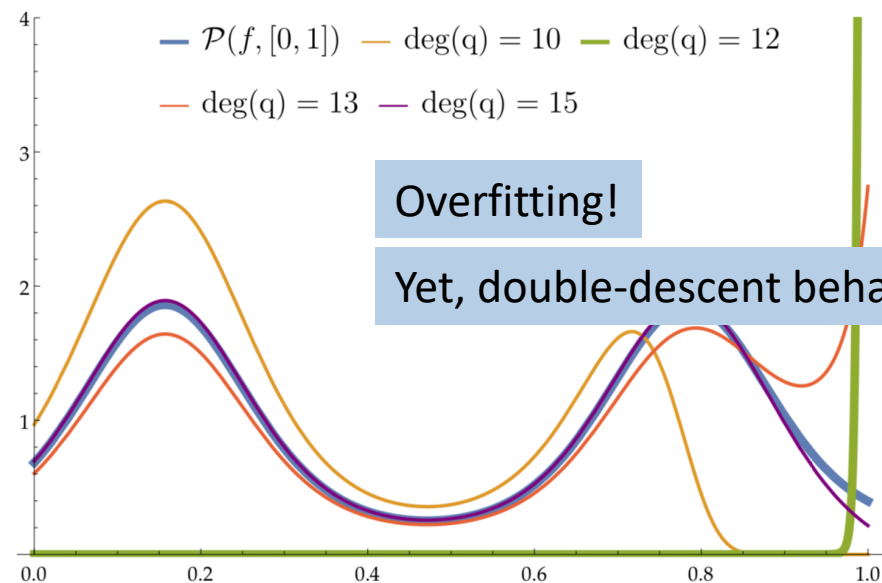
$$g(x) = \frac{e^{\sin 10x}}{\int_0^{1/2} e^{\sin 10x} dx}$$

(conditional of  $f(x)$  on  $[0, 0.5]$ )

**Experiment:** Take large sample  $S \subseteq [0, 0.5]^N$  from  $g(x)$ ; do MLE to fit most likely density  $\frac{e^{q(x)}}{\int_0^{1/2} e^{q(x)} dx}$ , where  $q$  is some polynomial.

**Question:** How well does fitted polynomial *extrapolate*?

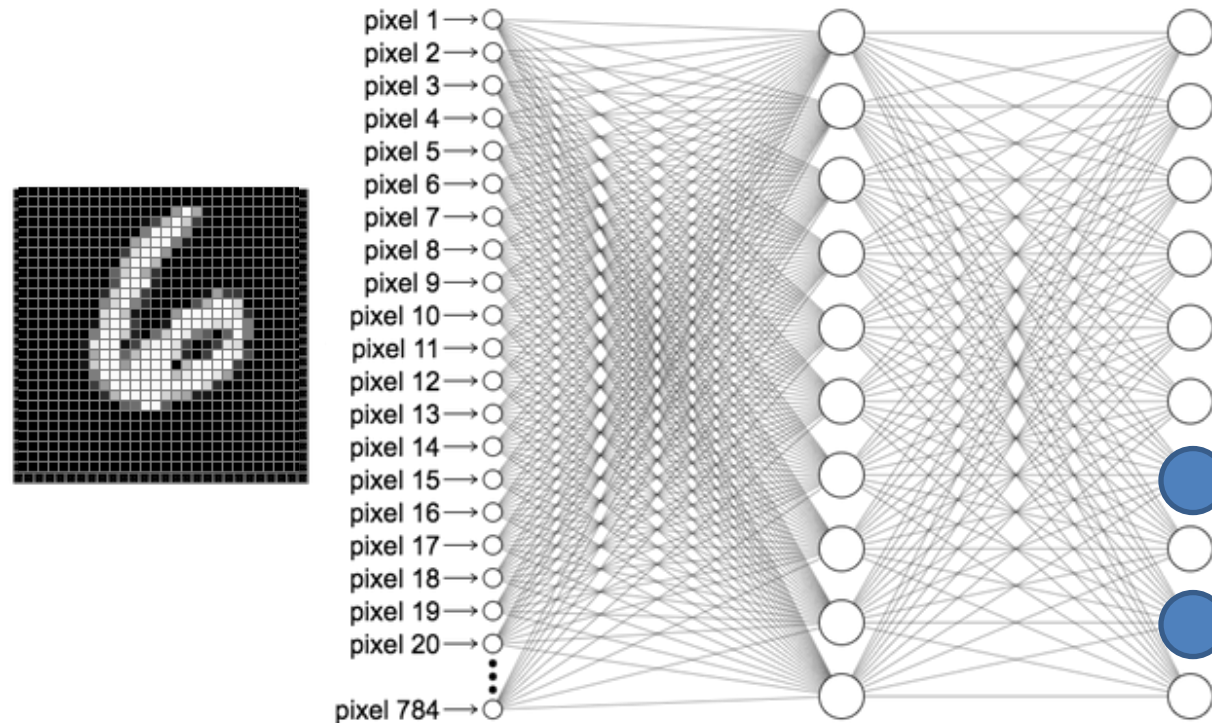
- compare  $\frac{e^{q(x)}}{\int_0^1 e^{q(x)} dx}$  to  $f(x)$



Overfitting!

Yet, double-descent behavior!

# Machine Learning Predictions

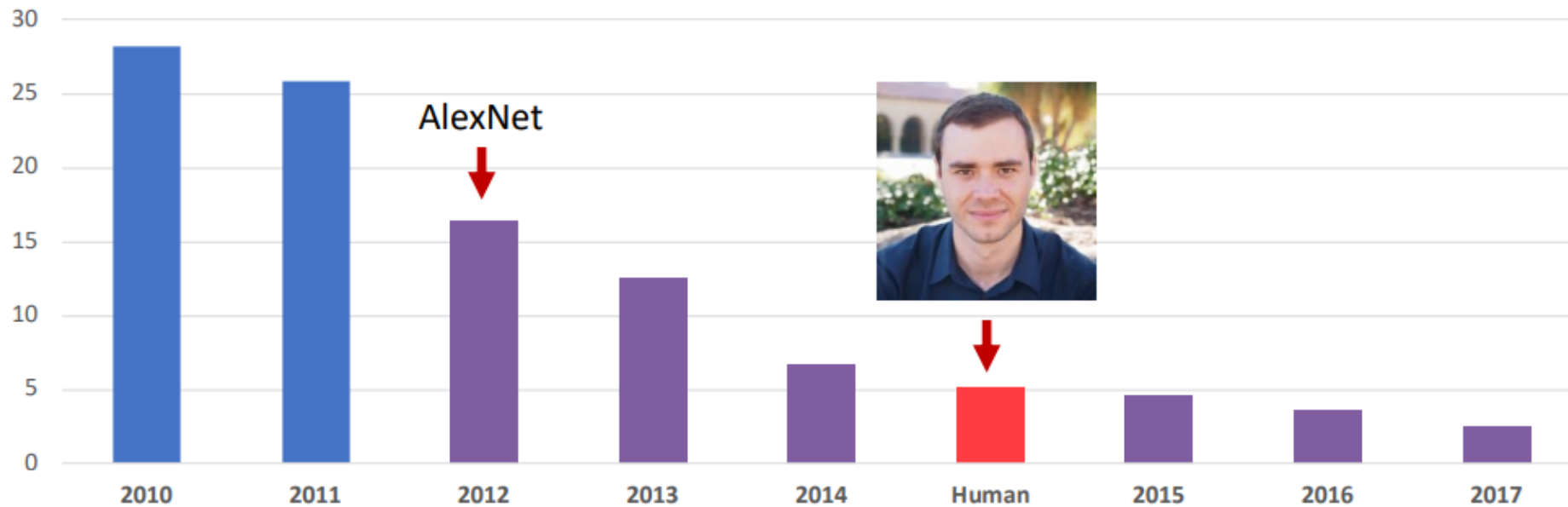


- Machine Learning Pipeline:
- Collect relevant data; partition into train set and validation set;
  - Train/choose hyperparameters
  - Deploy

# Machine Learning Predictions on Steroids



ILSVRC top-5 Error on ImageNet



But what do these results really mean?

# ProPublica AI Bias Article

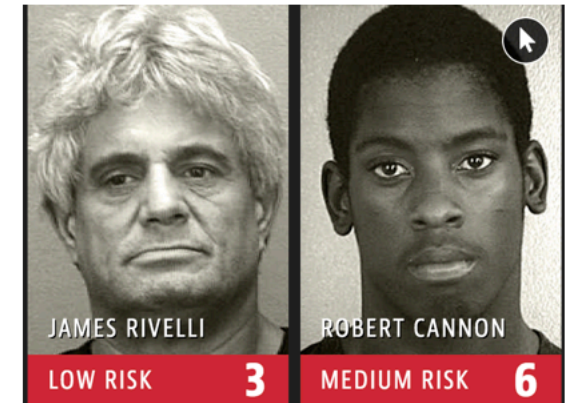
## Machine Bias

There's software used across the country to predict future criminals. And it's biased against blacks.

by Julia Angwin, Jeff Larson, Surya Mattu and Lauren Kirchner, ProPublica  
May 23, 2016

ON A SPRING AFTERNOON IN 2014, Brisha Borden was running late to pick up her god-sister from school when she spotted an unlocked kid's blue Huffy bicycle and a silver Razor scooter. Borden and a friend grabbed the bike and scooter and tried to ride them down the street in the Fort Lauderdale suburb of Coral Springs.

Just as the 18-year-old girls were realizing they were too big for the tiny conveyances — which belonged to a 6-year-old boy — a woman came running after them saying, “That’s my kid’s stuff.” Borden and her friend immediately dropped the bike and scooter and walked away.



JAMES RIVELLI	ROBERT CANNON
<b>Prior Offenses</b> 1 domestic violence aggravated assault, 1 grand theft, 1 petty theft, 1 drug trafficking	<b>Prior Offense</b> 1 petty theft
<b>Subsequent Offenses</b> 1 grand theft	<b>Subsequent Offenses</b> None
LOW RISK 3	MEDIUM RISK 6



# Executive Summary

## ❑ Selection bias in data collection

⇒ prediction bias (a.k.a. “ML bias”)

❑ **Goals:** decrease bias, by developing statistical methods robust to **censored and truncated samples**

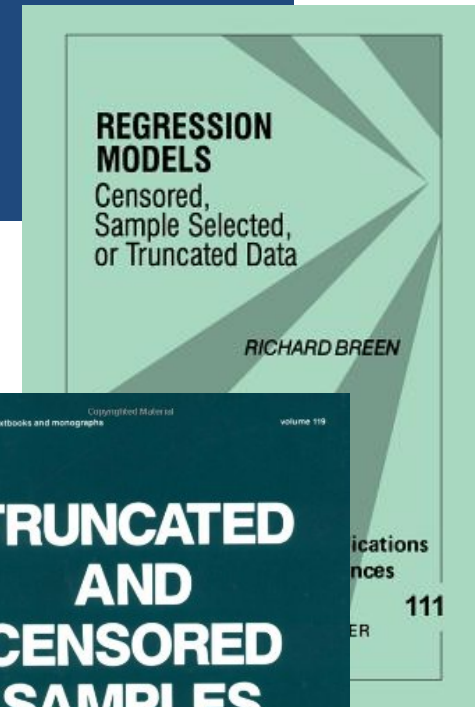
**Truncation:** samples falling outside of “observation window” are hidden and their count is also hidden

**Censoring:** ditto, but count of hidden data is provided

Why Censoring/Truncation?

- limitations of measurement devices
- limitations of data collection
  - experimental design, ethical or privacy considerations,...

- ❑ physics
- ❑ economics
- ❑ social sciences
- ❑ clinical studies



# Motivating Example: IQ vs Income

*Goal:* Relationship of IQ to Income for *low-skill workers* [Wolfe&Smith'56, Hause'71]

- “*low skill*” = paid under, say, \$10/hour

*Natural Approach:* survey families whose income is less than 1.5 times the poverty line; collect data  $(x_i, y_i)_i$  where

- $x_i$ : (IQ, Training, Education,...) of individual  $i$
- $y_i$ : earnings of individual  $i$

*Regression:* fit some model, e.g.  $y = \theta^T x + \varepsilon$

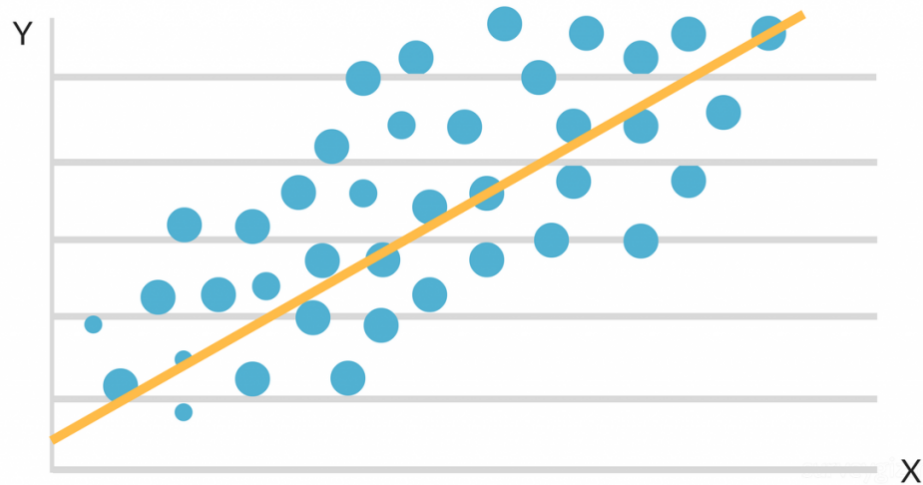
*Obvious Issue:* **thresholding incomes may introduce bias**

- it does, as shown by [Hausman-Wise'76] debunking prior results which had claimed that effects of education are strong, while of IQ are not

# What Goes Wrong in Presence of Truncation?

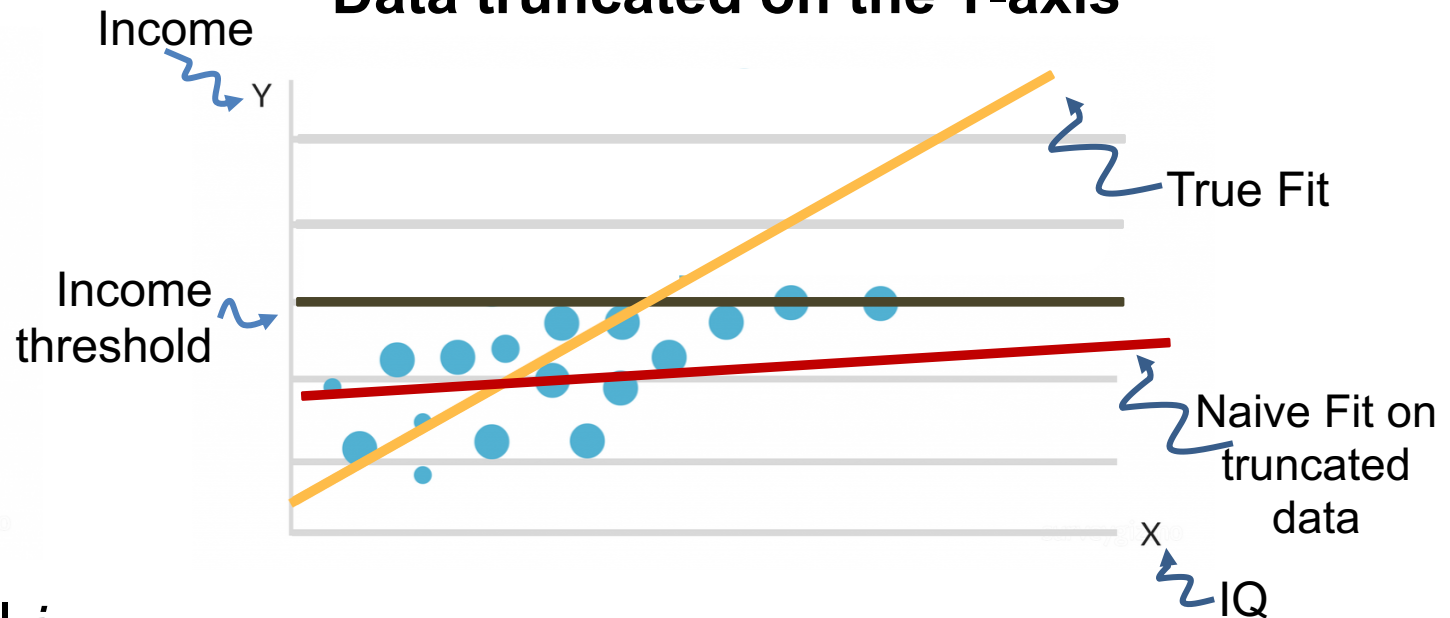
*Mental Picture:*

## Vanilla Linear Regression



Assumed truth:  $y_i = \theta \cdot x_i + \varepsilon_i$ , for all  $i$

## Data truncated on the Y-axis





















Supervised learning,  
with y-truncated data



Biased  
Models



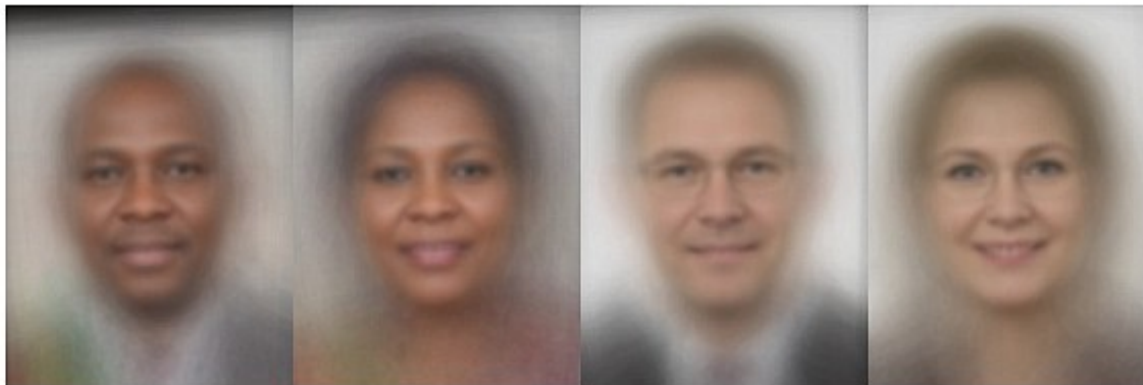
# Motivating Example 2: Gender Classification

Gender Classifier	Darker Male	Darker Female	Lighter Male	Lighter Female	Largest Gap
 Microsoft	94.0% 	79.2% 	100% 	98.3% 	20.8% 
 FACE++	99.3% 	65.5% 	99.2% 	94.0% 	33.8% 
 IBM	88.0% 	65.3% 	99.7% 	92.9% 	34.4% 

**Explanation:** Training data contains more faces that are of lighter skin tone, male gender, Caucasian

⇒ Training loss of gender classifier pays less attention to faces that are of darker skin tone, female gender, non-Caucasian

⇒ Test loss on faces that are of darker skin tone, female gender, non-Caucasian is worse



© MIT Media Lab

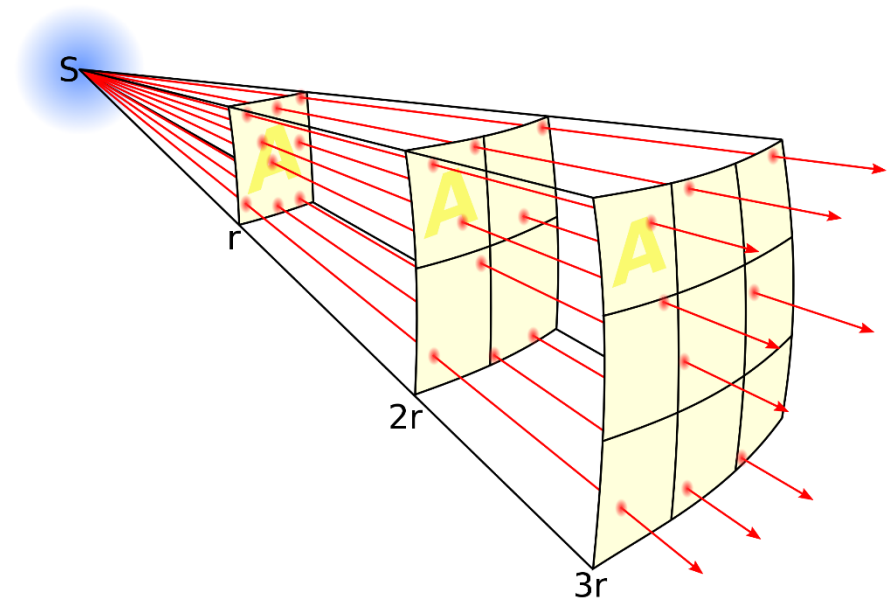
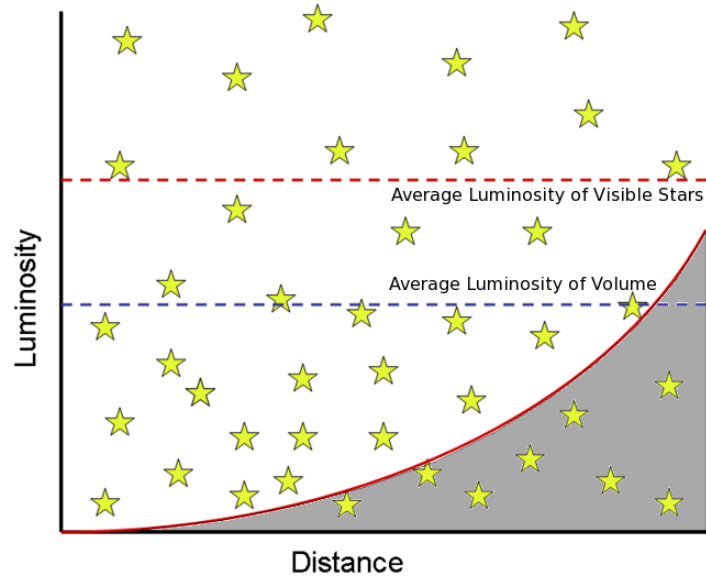
[Buolamwini, Gebru, FAT 2018]

Supervised learning,  
with x-truncated data



Biased  
Models

# Motivating Example 3: Malmquist Bias



Since light dims with distance, brightness limited surveys of the sky suffer from the cut-off of fainter objects at larger distances

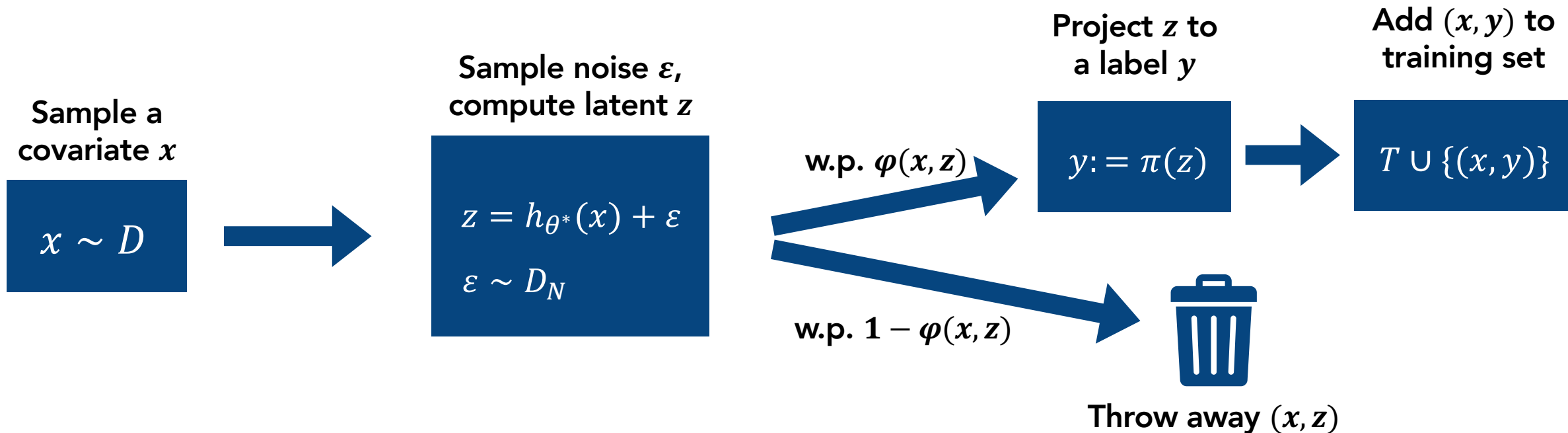
⇒ false trend of increasing intrinsic brightness, and other related quantities, with distance

Unsupervised learning,  
with truncated data



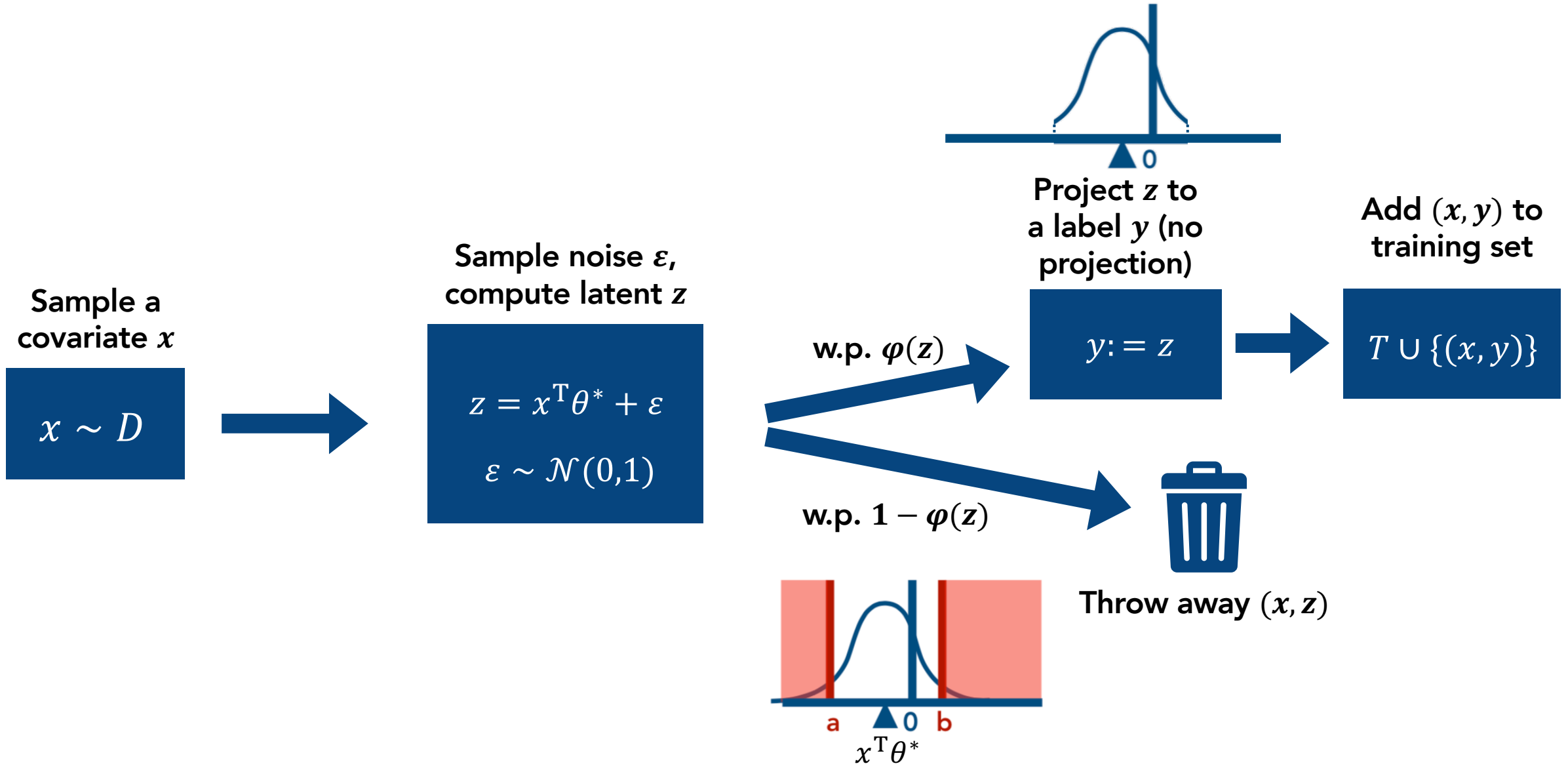
Biased  
Models

# Truncated Regression/Classification Framework



**Challenge:** Estimate  $\theta^*$  using training set  $T$  produced as above ( $\varphi$  is either known or from parametric family)

# e.g. Truncated Linear Regression



# e.g. Truncated Logistic Regression

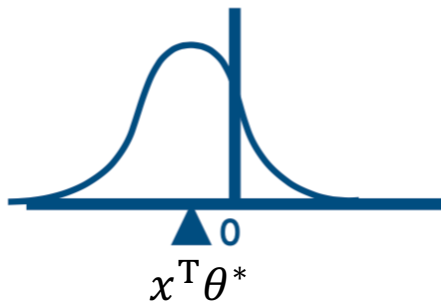
Sample a covariate  $x$

$$x \sim D$$



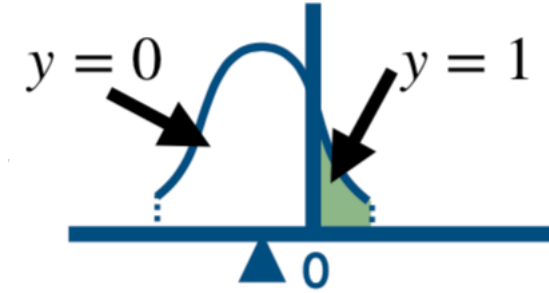
Sample noise  $\varepsilon$ ,  
compute latent  $z$

$$z = x^T \theta^* + \varepsilon$$
$$\varepsilon \sim \text{Logistic}(0,1)$$



w.p.  $\varphi(z)$

w.p.  $1 - \varphi(z)$



Project  $z$  to  
a label  $y$

$$y := 1_{z \geq 0}$$

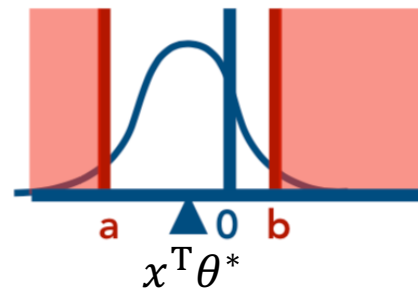


Add  $(x, y)$  to  
training set

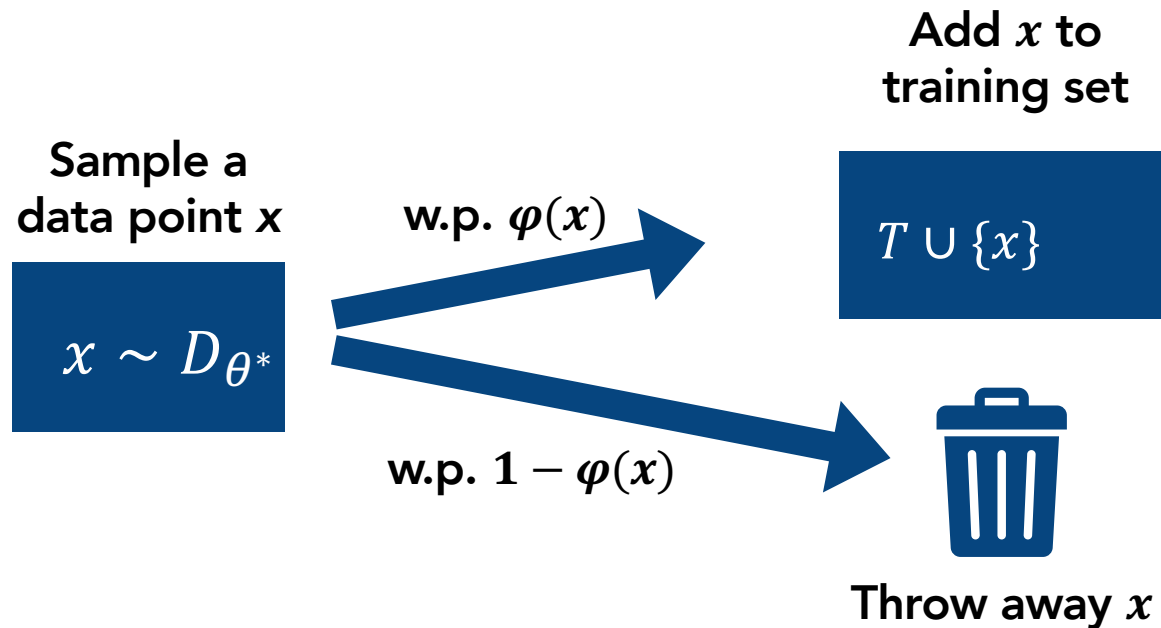
$$T \cup \{(x, y)\}$$



Throw away  $(x, z)$



# Truncated Density Estimation Framework



**Challenge:** Estimate  $\theta^*$  using training set  $T$  produced as above  
( $\varphi$  is either known or from parametric family)



# Censored/Truncated Statistics

How to train unbiased models from censored/truncated samples?

- Studied in Statistics/Econometrics since at least **[Bernoulli 1760]** **[Galton 1897]**, **[Pearson 1902]**, **[Pearson, Lee 1908]**, **[Lee 1914]**, **[Fisher 1931]**, **[Hotelling 1948]**, **[Tukey 1949]**, **[Tobin 1958]**, **[Amemiya 1973]**, **[Hausman, Wise 1976]**, **[Breen 1996]**, **[Hajivassiliou-McFadden'97]**, **[Balakrishnan, Cramer 2014]**, **Limited Dependent Variables models**, **Method of Simulated Scores**, **GHK Algorithm**
- Intimately related to domain adaptation in Machine Learning

Challenges:

- Error rates:  $\frac{\text{Bad}(d)}{\sqrt{n}}$
- Computationally inefficient algorithms

#parameters/dimension

#biased samples

Recent work **[w/ Gouleakis, Ilyas, Kontonis, Rohatgi, Tzamos, Zampetakis in FOCS'18, COLT'19, AISTATS'20, in progress]**

- Computationally and Statistically efficient algorithms; arbitrary truncation sets
- truncated linear/logistic/probit regression, compressed sensing, (non-parametric) density estimation
  - e.g. rates for linear regression  $O(\sqrt{d/n})$
  - e.g. rates for compressed sensing  $O(\sqrt{k \log d / n})$



# Censored/Truncated Statistics

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- Intimately related to domain

Challenges:

- Error rates:  $\frac{\text{Bad}(d)}{\sqrt{n}}$
- Computationally inefficient

Recent work [w/ Gouleakis, Ilyas,

- Computationally and Statistically
- truncated linear/logistic/probit regression, compressed sensing, (non-parametric) density estimation
  - e.g. rates for linear regression  $O(\sqrt{d/n})$
  - e.g. rates for compressed sensing  $O(\sqrt{k \log d / n})$

Why now?

- **Mathematics:** concentration/anti-concentration of measure [Carbery-Wright'01]
- **Machine Learning/Optimization:** stochastic gradient descent
- **Hardware:** gradient descent based algorithms exportable to Deep Neural Network models

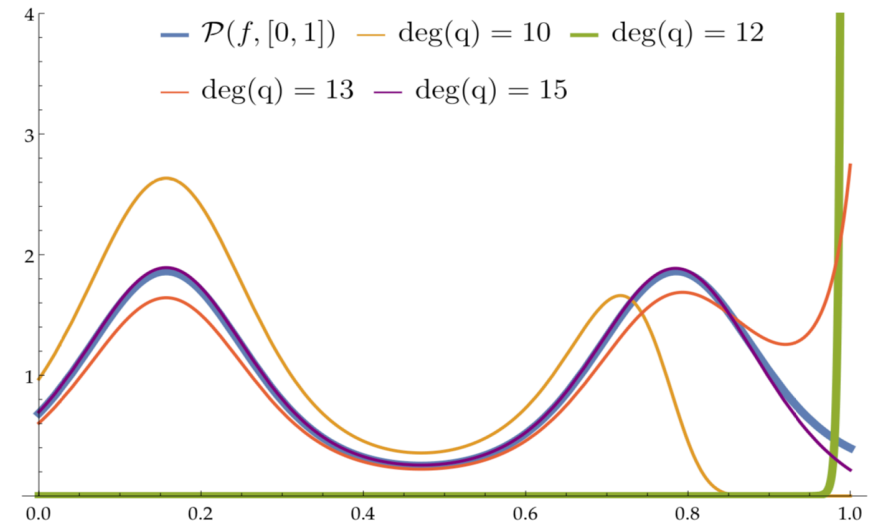
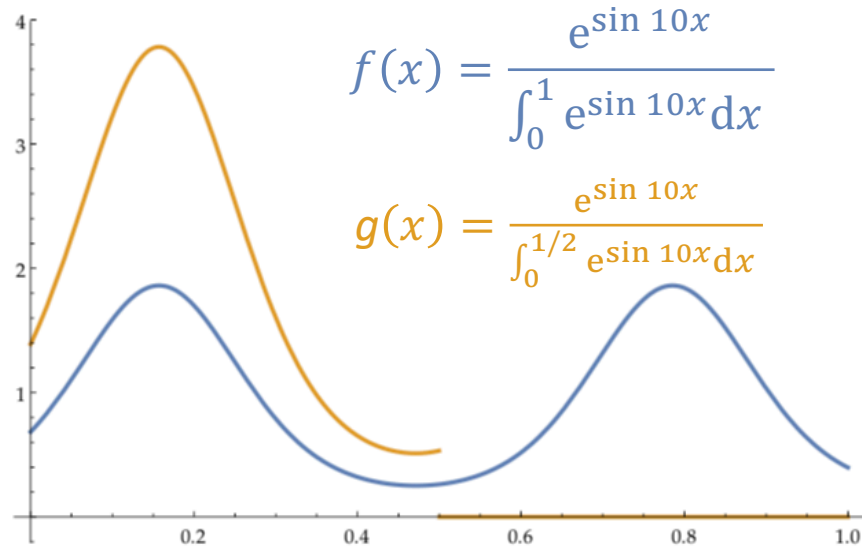
REGRESSION MODELS  
Censored,  
Selected,  
Truncated Data

RICHARD BREEN

Applications  
in  
Social Sciences  
111  
DISCUSSION PAPER

progress]

# When Does Extrapolation Work? (an impressionistic picture)



**Experiment:** Take large sample  $S \subseteq [0, 0.5]^N$  from  $g(x)$ ; do MLE to fit most likely density  $\frac{e^{q(x)}}{\int_0^{1/2} e^{q(x)} dx}$ , where  $q$  is some polynomial.

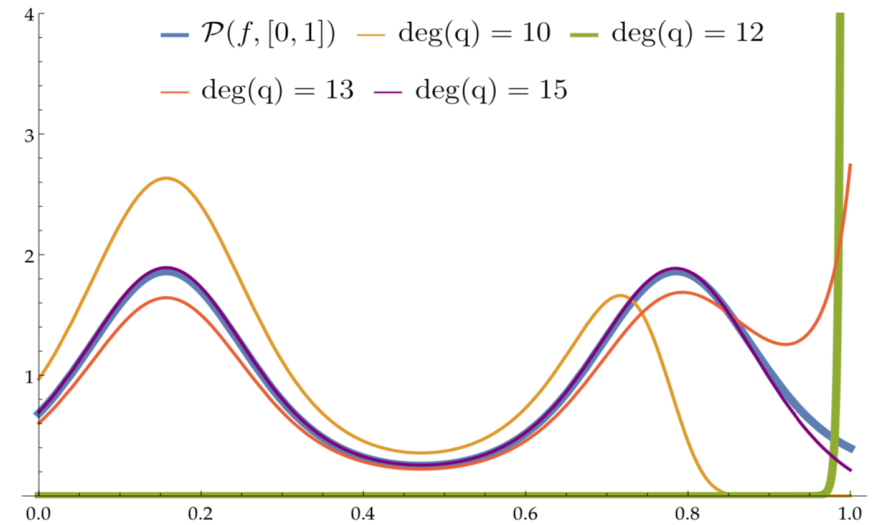
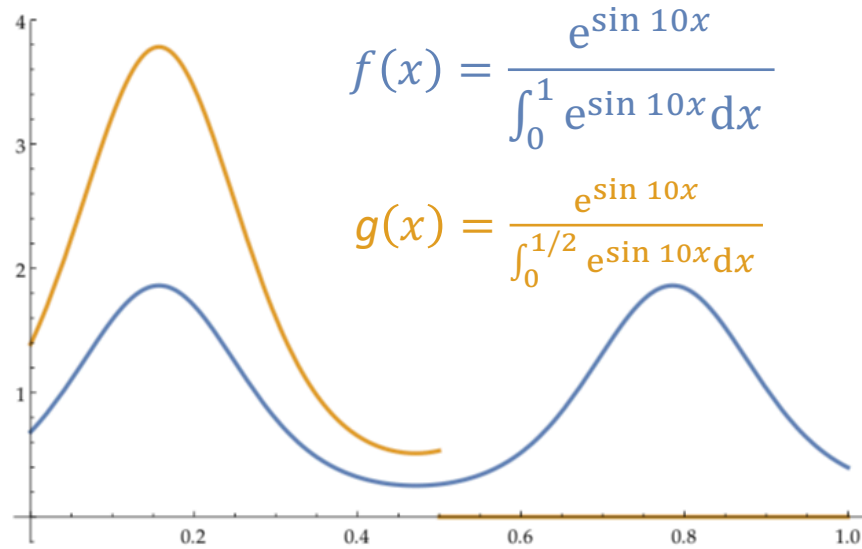
**Question:** How well does fitted polynomial *extrapolate*?

- compare  $\frac{e^{q(x)}}{\int_0^1 e^{q(x)} dx}$  to  $f(x)$

Overfitting!

Yet, double-descent behavior!

# When Does Extrapolation Work? (an impressionistic picture)



**Theorem:** Suppose  $P, Q$  are distributions over  $[0, 1]^d$ , whose log-densities are polynomials of degree  $k$ . Suppose  $S \subseteq [0, 1]^d$  has  $\text{vol}(S) \geq \alpha$ . Then:

$$\left(\frac{d}{\alpha}\right)^{-O(k)} \leq \frac{TV(P, Q)}{TV(P_S, Q_S)} \leq \left(\frac{d}{\alpha}\right)^{O(k)}$$

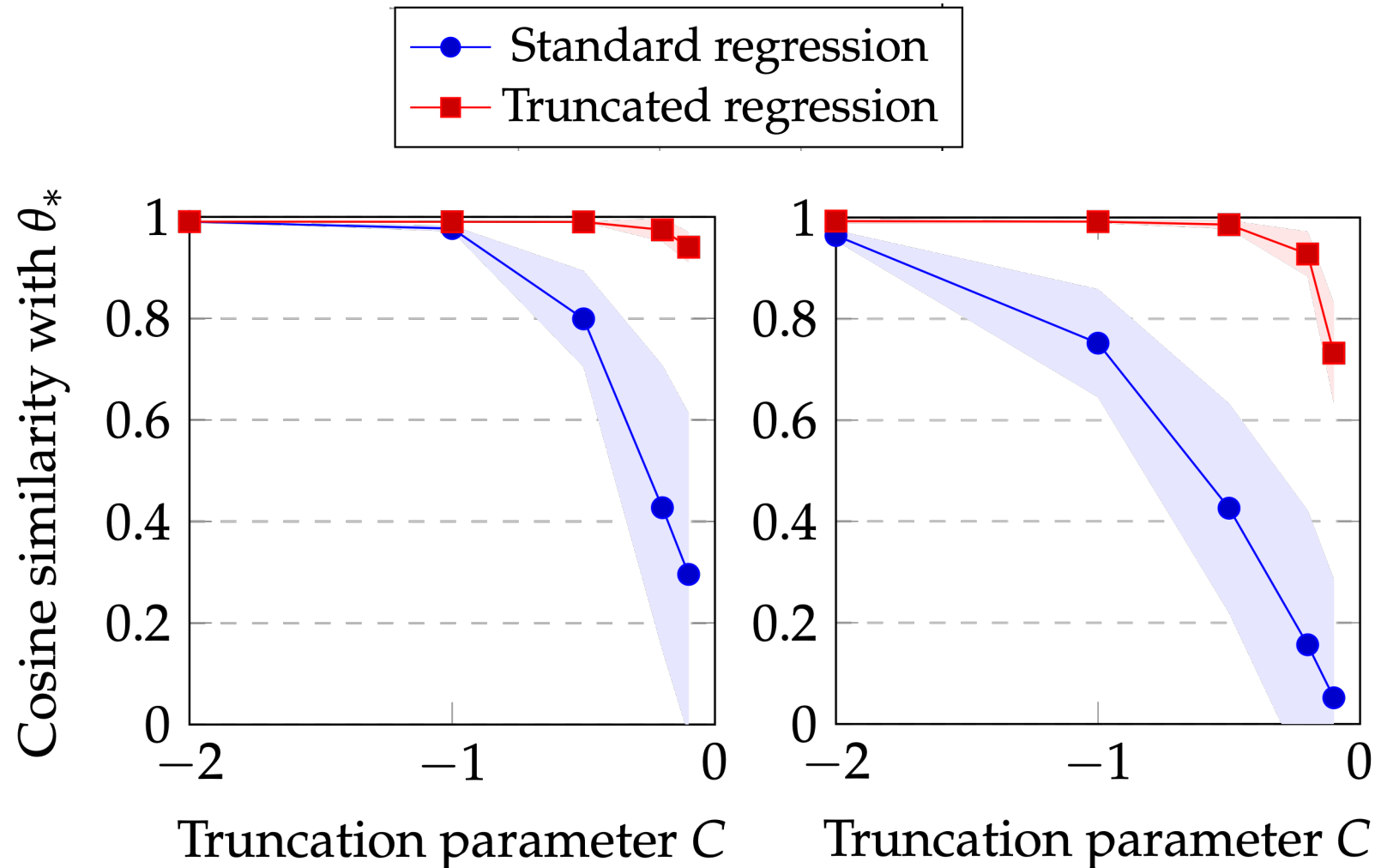
**Implication:** If  $P, Q$  are far in their whole domain, their conditionals can't appear too close.

# Experiment: Logistic and Probit Regression

## Synthetic data

### Setup:

- $\theta^* \sim \mathcal{U}([-1,1]^{10})$
- $X_1, \dots, X_n \sim \mathcal{U}([0,1]^{10})$
- $Z_i := \theta_*^\top X_i + \varepsilon_i$
- $\varepsilon_i \sim \mathcal{N}(0,1)/\text{Logistic}(0,1)$
- Truncation:  $\varphi(\cdot) = \mathbf{1}_{[C, \infty)}$
- Projection:  $Y_i = \mathbf{1}_{Z_i \geq 0}$ 
  - when  $C = 0$  only see positive examples

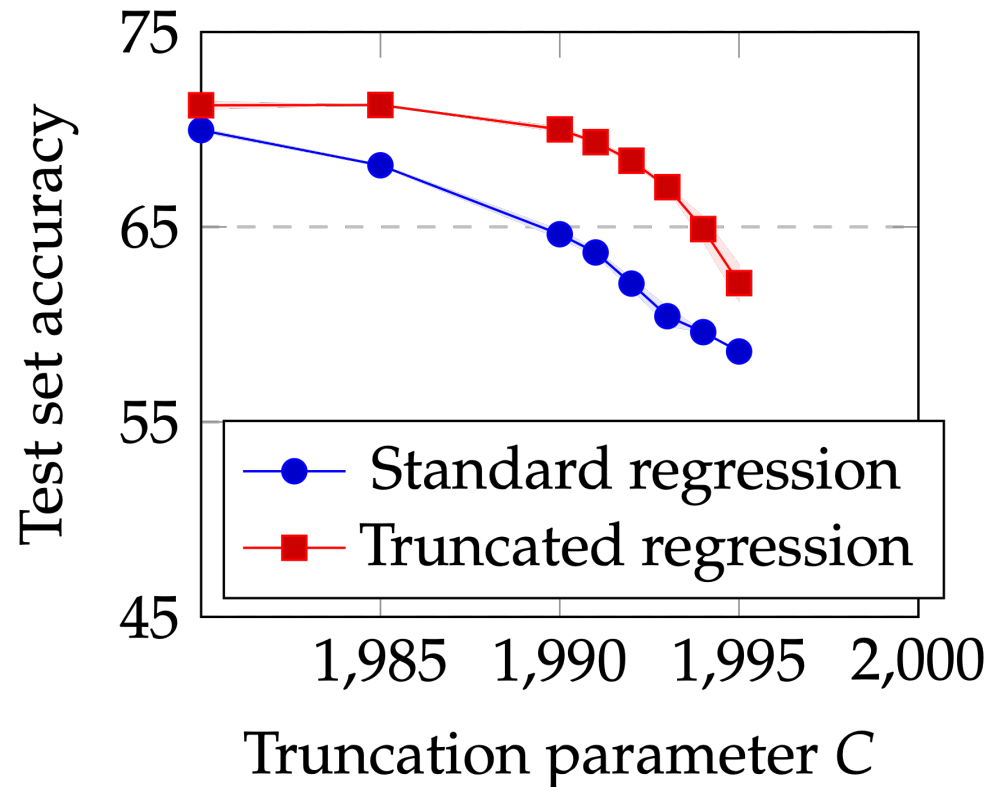


# Experiment 2: Logistic Regression

## UCI MSD dataset

### Setup:

- $X$ : song attributes
- $Z$ : year recorded
- Truncation  $[C, \infty)$
- $Y$ : recorded before '96?





# Experiment 3: Extreme Domain Adaptation

Train Set



Test Set



Metaphor of settings where support of test set distribution is measure 0 on support of train set distribution

Test Error of Naïve AlexNet Gender Classifier: 55%

Improvement using truncated Statistics: 80%

# Conclusions

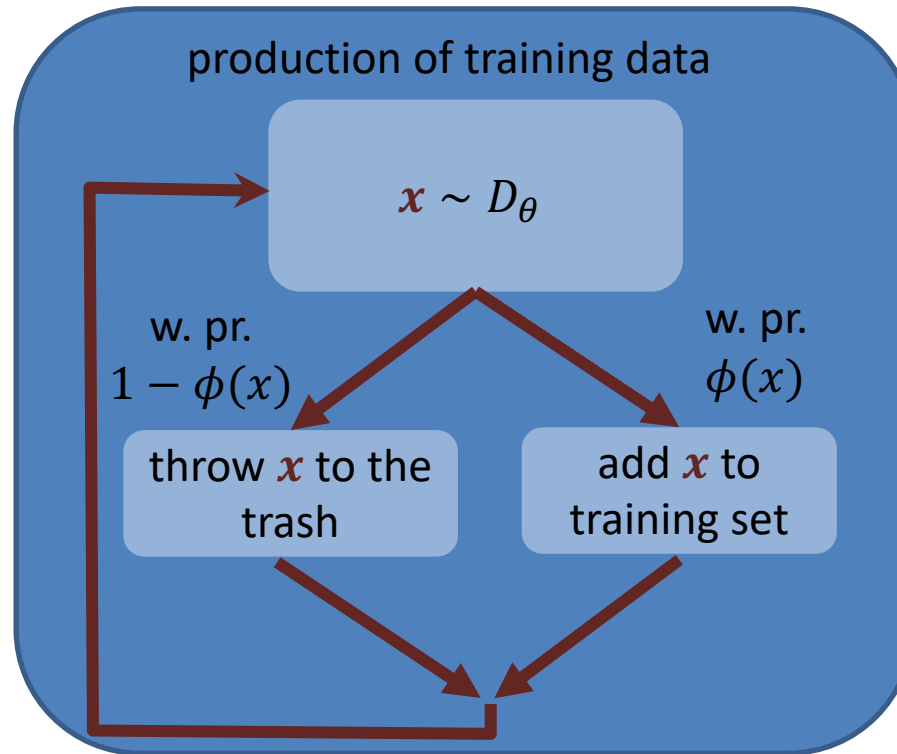
- ❑ **Missing Observations**  $\Rightarrow$  prediction bias (a.k.a. “AI bias”)
- ❑ **Our Work:** decrease bias, by developing statistical methods more robust to **censored and truncated samples**
- ❑ **General Framework:** SGD on Population Log-Likelihood (applies to DNNs)
- ❑ **End-to-end guarantees:** statistical rates and efficient algorithms for several classical problems in Statistics: linear/probit/logistic regression, compressed sensing, non-parametric density estimation
- ❑ **Future work:** push further on reducing parametric assumptions

Thank you!

- Skipped Slides

# Censored/Truncated Statistics

## Truncated Density Estimation



**Goal:** Estimate  $\theta$  using truncated training set